Individual hip prosthesis design from CT images

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Abstract: Before surgery, a sequence of transaxial hip CT images of the patient is scanned. Using these images, our method computes the shape of the prosthesis stem which is optimal with respect to the femoral anatomy. The stem is modelled by a generalized cylinder. The optimal 3-D shape is fitted using a sequential procedure of two-dimensional least-squares smoothing subject to constraints. For each individual, this procedure is fully automatic.

Key words: General cylinder fitting, medullary canal shape analysis, individual hip prosthesis, CT images sequence.

Introduction

Hip prosthesis is composed of a sphere, the femoral head, screwed on a stem which the surgeon hammers in the medullary canal of the femur (Figure 1). There are only few available stems at the surgeon’s disposal. He chooses among these on the basis of an anterior-posterior hip radiograph, so that this choice is sometimes hazardous. Thus, with conventional methods, the stem fits only roughly in the canal – the gaps between the stem and the femoral wall are filled with ciment – and consequently, there are some risks of loosening for the prosthesis and of weakening for the whole set-up.

The aim of this paper is to construct a computerized sequence of processes leading to an exact fitting of the hip prosthesis to each individual’s anatomy. Such a prosthesis is called Individual Computer Designed (ICD) prosthesis. The method is partly described by Chalmond (1985). A preliminary design is given by Cinquin (1982). This author’s aim is to recover the shape of the canal itself from hip radiographs, in order to deduce afterwards a design for the stem. Recently, Granholn (1987), has given a partial solution to our problem. They have developed a series of interactive software programs for designing an optimal fit custom hip stem based on 3-D bone geometry data obtained from CT scans. Thirty individual femoral models were generated, scaled to a standard size, and their geometry averaged. This averaged shape was then used to produce a stem which was scaled to produce five standardized sizes. These stems are now undergoing clinical trials. We think that this method does not provide ICD prostheses but rather a computer designed prosthesis which is only adapted to the averaged morphology of the individuals. But the shape of the individual medullary canals can vary very much from this average as we shall show later (Figure 6). So, we claim that the only stem which can achieve an optimal fitting must be adapted to each individual’s anatomy.

In here, we directly give a design for the stem. The femoral head level and angle can easily be calculated from this design. The anatomical fitting of the stem needs four steps:

(a) a sequence of hip CT images is obtained according to a prescribed protocol;
(b) each digitized image is processed to perform a sampling of the medullary canal boundary;
(c) from these samples, we compute the optimal shape of the stem under several constraints;
Optimal shape design

On each image, \( m \) points are extracted from the canal boundary. The data is thus composed of \( n \) samples of \( m \) values lying in \( n \) parallel planes. Our aim is to determine by smoothing a parametric shape which fits this data. Several classical methods could perform such a smoothing (e.g. Computer Aided Design, see Pratt (1985) for a recent survey). However the shape of the stem must satisfy several constraints that the above methods do not take into account. The first constraint is obvious: the stem must be surgically insertable in the canal and fit in very tightly. Next, the shape must be of simple geometry while respecting the canal features.

This section has four paragraphs. The first one gives the general principle of the optimal shape design. It relies on a least-squares criterion which is described in the second paragraph. In the third paragraph, this shape is reduced in order to be wholly inside the canal. The last paragraph explains the grounds for this design.

1. Design and method

A generalized cylinder is chosen as a model for the stem. By definition, this is a 3-D geometric surface characterized by a 3-D curved axis on which closed cross-section curves develop perpendicularly and continuously (see Shani and Ballard (1984)). This model is difficult to adapt to 3-D data. In our
case, this difficulty is attenuated because our 3-D data correspond to parallel cross-sections. Near the lesser trochanter, the canal boundary on coordinates \((z_1, \ldots, z_n)\), can be erratic while on \((z_{q+1}, \ldots, z_n)\) it looks like an ellipse (Figure 3). Let us now describe our method of fitting; it is a sequential procedure with four steps:

1. Each cross-section sample is smoothed into an ellipse.

2. The \(n\) centers of these ellipses define a sample of the canal axis. The projections of these centers in the planes \(Oxz\) and \(Oyz\) are separately smoothed by a piecewise polynomial function of degree 3 if \(z \leq z_q\) and degree 2 if \(z > z_q\). Both functions are continuous up to the first derivative. They define the stem axis.

3. The first step is again performed, subject to the constraint that the ellipses must be centered on the stem axis.

4. Each ellipse is characterized by its center \((X, Y)\), its two axes \(d, D\) and the angle \(A\) between its great axis and \(Ox\). The \(n\) values of \(d, D, A\) are separately smoothed as in step (2).

The resulting generalized cylinder is composed of a curved axis whose projections on \(Oxz\) and \(Oyz\) are polynomial curves:

- for \(X(z) = a_1^{(k)} + b_1^{(k)}z + c_1^{(k)}z^2 + d_1^{(k)}z^3\),
- for \(Y(z) = a_2^{(k)} + b_2^{(k)}z + c_2^{(k)}z^2 + d_2^{(k)}z^3\), \(\forall z_1 \leq z \leq z_n\),

and of ellipses centered on this axis defined by:

- \(d(z) = a_3^{(k)} + b_3^{(k)}z + c_3^{(k)}z^2 + d_3^{(k)}z^3\),
- \(D(z) = a_4^{(k)} + b_4^{(k)}z + c_4^{(k)}z^2 + d_4^{(k)}z^3\),
- \(A(z) = a_5^{(k)} + b_5^{(k)}z + c_5^{(k)}z^2 + d_5^{(k)}z^3\), \(\forall z_1 \leq z \leq z_n\),

with \(k = 1\) if \(z \leq z_q\) and \(k = 2\), \(d_1^{(2)} = d_2^{(2)} = d_3^{(2)} = d_4^{(2)} = d_5^{(2)} = 0\) if \(z > z_q\). \(k\) indicates which part of the stem we are in.

2. Mathematical tools

(a) Fitting a conic to a set of points in the plane.

We follow the scheme detailed in Bookstein (1979) and Gnanadesikan (1979). Given \(m\) points \(\{(x_i, y_i) : i = 1, \ldots, m\}\) in the plane, we estimate the coefficients of the equation \(Q(x, y) = ax^2 + bxy + cy^2 + dx + ey + f = 0\), by minimizing \(\sum_i Q(x_i, y_i)\) with respect to \(w = (a, b, c, d, e, f)\) subject to \(w'Kw = \text{constant}\) where \(K\) is a diagonal matrix. If, in addition, we require the conic to be centered at \((x_0, y_0)\), then the coefficients must also satisfy: \(2ax_0 + by_0 + d = 0\) and \(bx_0 + 2cy_0 + e = 0\). The solution is given in Rao (1973).

(b) Fitting a continuous piecewise polynomial function.

Let us consider for instance the \(D\) axis. We have to estimate the coefficients of:

- \(D(z) = a_1^{(k)} + b_1^{(k)}z + c_1^{(k)}z^2 + d_1^{(k)}z^3\)

with \(k = 1\) if \(z_1 \leq z \leq z_q\) and \(k = 2\), \(d_1^{(2)} = 0\) if \(z_q \leq z \leq z_n\), given \(\{(z_i, D_i) : i = 1, \ldots, n\}\) by the least-squares criterion. This is the classical problem of
spline functions except that the degrees of the polynomials are not constant. The linear model is:

\[ D_t = a^{(1)} + b^{(1)} z_t + c^{(1)} z_t^2 + d^{(1)} z_t^3 + \varepsilon_t \quad \text{if} \quad 1 \leq t \leq q, \]

\[ D_t = a^{(2)} + b^{(2)} z_t + c^{(2)} z_t^2 + \varepsilon_t \quad \text{if} \quad q < t \leq n \]

where \( \{\varepsilon_t\} \) are centered random variables. Recall that \( D \) is continuous up to the first derivative. From the continuity, we have:

\[ a^{(1)} + b^{(1)} z_q + c^{(1)} z_q^2 + d^{(1)} z_q^3 = a^{(2)} + b^{(2)} z_q + c^{(2)} z_q^2, \]

\[ b^{(1)} + 2c^{(1)} z_q + 3d^{(1)} z_q^2 = b^{(2)} + 2c^{(2)} z_q. \]

The least-squares estimate minimizes \( \sum \varepsilon_t^2 \) subject to the continuity constraints.

3. Reduction

The least-squares criterion constructs a shape which is not wholly inside the canal. We shall obtain a shape which is nearly completely inside the canal. Let \( Q_t(x, y) = 0 \) denote the fitted ellipse in the \( z_t \) plane, \( t \in [1, \ldots, n] \); its axes \( d(z_t) \) and \( D(z_t) \) are defined by the fitted generalized cylinder in \( \S 1 \). When \( (x_t, y_t) \) is inside the ellipse, then \( Q_t(x_t, y_t) < 0 \) otherwise \( Q_t(x_t, y_t) \geq 0 \). Now, we describe a method for reducing \( d(z) \) and \( D(z) \), \( z \in [z_1, z_n] \), so that the reduced ellipses, say \( Q'_t \), satisfy:

\[ \forall t, \forall i: \quad Q'_t(x_i, y_i) > \eta \]

where \( \eta \) is a small negative value ensuring a tight fitting (for instance \( \eta = -0.08 \)). This method has two steps:

**Step 1.** For each cross-section sample, \( d(z_t) \) and \( D(z_t) \) are separately reduced. Let \( r(z_t) \) and \( R(z_t) \) be the reduction coefficients. They are the outcome of a simple process which gradually reduces \( d(z_t) \) and \( D(z_t) \) by subtracting a small value \( \zeta \) to the current axis values, so that the resulting reduced ellipses, say \( Q'_t \), satisfy: \( \forall i, Q'_t(x_i, y_i) > \eta \), as follows. At the \( j \)th iteration, the process calculates:

\[ M_j^d = \text{Min}_i\{Q_i^{(1,0,0)}(x_i, y_i)\} \quad \text{and} \quad M_j^D = \text{Min}_i\{Q_i^{(1,0,0)}(x_i, y_i)\} \]

where \( Q_i^{(1,0,0)} \) is the ellipse whose axes are \( d_{j-1}(z_t) - \zeta \), \( D_{j-1}(z_t) \) and \( Q_i^{(1,0,0)} \) whose axes are \( d_{j-1}(z_t) \), \( D_{j-1}(z_t) - \zeta \) (\( d_0 = d \), \( D_0 = D \)). If \( M_j^d > M_j^D \), then \( D_{j+1}(z_t) = D_{j-1}(z_t) - \zeta \), \( d_{j+1}(z_t) = d_{j-1}(z_t) \) otherwise \( D_{j+1}(z_t) = D_{j-1}(z_t) \), \( d_{j+1}(z_t) = d_{j-1}(z_t) - \zeta \). If \( M_j^d > \eta \) or \( M_j^D > \eta \), the process ends and we set:

\[ R(z_t) = D(z_t) - d(z_t), \quad r(z_t) = d(z_t) - d(z_t). \]

**Step 2.** At the end of the first step, we get two series: \( \{r(z_t): t = 1, \ldots, n\} \), \( \{R(z_t): t = 1, \ldots, n\} \) which are now separately smoothed by a continuous piecewise polynomial function as in the previous paragraph without the constraint \( d^{(2)} = 0 \), which yields two curves \( S(z) \) and \( s(z) \), \( z \in [z_1, z_n] \). Let \( e = \text{Max}_i\{r(z_i) - s(z_i)\}, \quad E = \text{Max}_i\{R(z_i) - S(z_i)\} \). Then, axes \( d \) and \( D \) are reduced according to:

\[ d(z) - [s(z) + e], \quad D(z) - [S(z) + E]. \]

So, we have constructed a smooth and non-uniform shape reduction which is small, if the ellipse is well fitted, and great otherwise.

4. Insertability of the design

Now, we have to check that the designed shape can be inserted in the canal. Near the lesser trochanter, the boundary may be confuse and even not defined in some regions. For this reason, the stem is composed of two continuously connected pieces. But, in the lower part of the femur, \( (z_q \leq z \leq z_n) \), the canal morphology indicates that a generalized cylinder can be inserted when its axis coordinates are

![Figure 4. Lower views of an ICD stem: (a) with angular sectors. (b) with great axes.](image-url)
parabolic branches without extremum and when its cross-sections are ellipses with decreasing areas. Mostly, these conditions are achieved by the fitted generalized cylinder. But if it is not, the steps (2) and/or (4) of the procedure must be modified by considering the following constraints: on \([z_a, z_n], X, Y, d\) and \(D\) being modeled by a quadric \(a^{(2)} + b^{(2)}z + c^{(2)}z^2\), we shall request that \(z_{\text{opt}} \notin [z_a, z_n]\) where \(z_{\text{opt}} = -b/2c\) is the quadric optimum abscissa. That leads us to add a new constraint in §2(b): 
\[2c^{(2)}z_n + b^{(2)} > 0 \text{ or } 2c^{(2)}z_n + b^{(2)} < 0.\]
Then, the procedure is again performed.

Results and comments

We comment now on the figures. In Figure 3, a partial cross-section series of a fitted shape before and after reduction is displayed. Figure 4 shows a fitted shape which is projected on its basis. In Figure 4(a), the curves from the lower to the upper ellipse correspond to angular sectors: in Figure 4(b), the straight segments are the great axis \(D(z_e)\) of the ellipses, they illustrate the torsion of the canal. Furthermore, the curve CD represents the projected axis of the generalized cylinder. The Figures 5(a) and 5(b) are two perspective views of a fitted shape which are truncated to clarify the view. Figure 5(b) only shows the sections corresponding to the \(n\) samples; Figure 5(a) displays the continuous shape. Figure 6 is displayed as Figure 4 and shows two ICD shapes which are projected on the same plane. They illustrate the difference between the individual morphologies. This fact proves that standardized custom stems cannot provide good fitting for individual bones. Figure 7 presents the photography of a ICD metal stem which was obtained by milling. Figure 8 comes from a digitized anterior-posterior radiograph of a total ICD hip replacement. We notice an exact fitting along the proximal medial wall of the femur. This fitting and the torsion fitting together help to achieve axial torsional stability for the stem.

Notice that throughout the design, we argued on the basis of the \(n\) samples of the canal which are obtained at step (b). But during the operation, the medullary canal is removed by reaming and rasping and so the sections of the resulting cavity do not look exactly like the samples. Yet, the stem finds naturally its place in the cavity during the hammering. This is partly due to the torsional shape of the stem (Figures 4, 5). In the near future, ICD rasps will be constructed with each ICD stem by using a
modified version of the stem model. Let us add that
the knowledge of the stem parameterized axis helps
to calculate the appropriate position and the orien-
tation of the femoral head. Several clinical trials
have been led by Prof. P. Grammont and Dr. P.
Trouilloud of the Hospital Medical School in
Dijon. Up to now, seven cimentless hip replace-
ments by ICD prostheses have been successfully ac-
complished on selected cases. These prostheses are
commercially available and are undergoing clinical
trials in several Hospitals Centers in France.

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